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STATISTICS

MICROCOMPUTER GRAPHICS FOR STATISTICAL EDUCATION

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Abstract—Statistics pervades all facets of our lives and is one of the most important courses for non-science or non-engineering majors. The microcomputer frees students and teachers from much of the drudgery of numerical calculations, while providing the graphics to enhance understanding. This article demonstrates how microcomputer graphics can be used to do such things as instill the concept of randomness, make plausible the convergence of the binimial to the normal distribution, and give convincing visual displays of the Central Limit Theorem.

INTRODUCTION

In recent years, increasing availability and decreasing cost have brought microcomputers into the mathematics classroom in growing numbers. Computers are being used widely and successfully in the CAI mode in algebra and other remedial classes. In the graphics mode, computer usage is rapidly growing in precalculus, calculus and differential equations to enhance student understanding of topics and methods through visual impact.

At the same time, statistics is fast becoming the single most important mathematics offering for the non-mathematics, science or engineering major. Several prominent groups concerned with mathematics education have called for statistical literacy to be an essential component of a modern college education. This reflects the way that statistics now pervades almost all facets of our lives. Moreover, most of the tremendous growth in statistics and its applications over the last 30 years or so is directly attributable to the availability of high speed computers.

Therefore, the next step would seem to be a fairly extensive use of computers in statistics courses. To this point in time, however, this has not materialized to any great extent. This may be due to the almost universal adoption of hand-held calculators in such courses. It is only very recently that there has been any indication of a slowly growing trend toward the use of computers by statistics students to perform statistical computations using prepared packages such as SPSS, Minitab, BMD, etc., especially in mainframe computer environments. Further, a variety of publishers are beginning to come out with more student-oriented microcomputer packages for elementary and applied statistics. Unfortunately, especially at the elementary level, the result is usually little more than having the computer serve as a glorified hand-held calculator. This is certainly desirable in terms of eliminating most of the drudgery associated with doing the computations by hand. However, it is merely one small facet of the enormous potential for computer usage in statistical education.

The typical elementary (non-calculus) based statistics course emphasizes the concepts and methods of inferential statistics rather than the computations. Similarly, a solid calculus-based introductory statistics course emphasizes the concepts and the theory over the computations. Unfortunately, in the existing statistical software packages, virtually no attention has been paid to the graphical representation of the concepts of statistics. Yet, modern microcomputer graphics provides a powerful tool to transform most of the statistical concepts and methods into visual representations which dramatically improve student comprehension. Most business-oriented software packages already provide the capability of displaying data in visual form via histograms, bar charts and pie charts. Nonetheless, this represents little more than the first week of the standard introductory statistics course.

MICROCOMPUTER GRAPHICS FOR INFERENTIAL STATISTICS

Introductory statistics (both calculus and non-calculus based courses) can use the power of computer graphics for simulations and demonstrations of most of the important concepts and methods of inferential statistics. Most of the programs discussed in the present article have been written by the authors for the IBmPC, the Apple II series and various Radio Shack TRS80 models. The range of possible uses of the computer can go from very simple demonstrations up to complex simulations. The unifying motive is to increase student understanding of the statistical concepts.

The underlying concept that pervades all of statistics is that of randomness. It is a particularly deceptive topic in the sense that few students truly grasp what randomness is all about. This problem is not always evident in typical classroom situations, but becomes painfully obvious if students are required to collect and analyze random data. They have no feel for what it means to make a random selection. The use of a table of random numbers seems little more than a totally unnecessary complication or nuisance to them.

The computer can provide a major asset in conveying some feeling for randomness in statistics. At a simplistic level, it can be used to generate repeated random selections from a set of pre-selected numbers or random selections of Heads and Tails to simulate coin-tossing experiments. However, through the power of computer graphics, students can achieve a much more effective visual demonstration of random phenomena.

Possibly the simplest and most accessible (to the student) application of randomness occurs in a two-dimensional random or drunkard's walk problem. As illustrated in Fig. 1, the random path traversed is drawn for any choice of step size and number of such steps. Figure 2 shows the completely different path that results for the same choice of step length and number of steps. Students are usually stunned by the fact that the resulting paths can be so totally dissimilar. Their reaction clearly underscores the authors' conviction about their complete lack of comprehension of random phenomena. At an initial level, a program such as this gives the student a visual feel for randomness. In a more advanced course, the random walk phenomenon can be effectively tied to a variety of important and interesting processes.

In a related direction, the concept of randomness can also be approached effectively through the binomial distribution which can be simulated extremely effectively via a coin-tossing experiment. Numerous people have written such programs in the purely numerical mode. Typically, in such a program, the user will choose the number N of fair coins and the program will randomly select

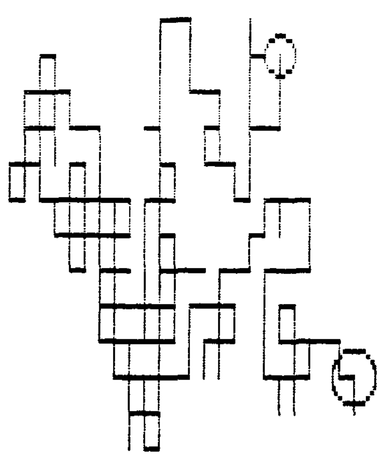

Fig. 1. Two-dimensional random walk simulation. Random walk process with 300 steps of length 7.

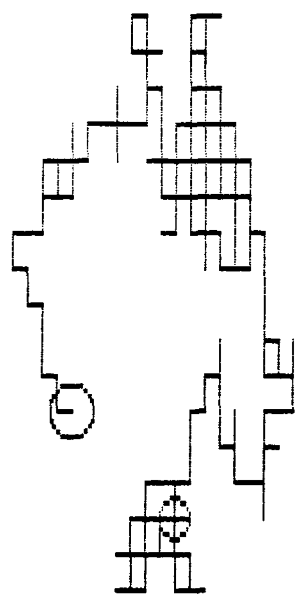

Fig. 2. Two-dimensional random walk simulation—second path. Random walk process with 300 steps of length 7.

sets of N coins, display the appropriate sets of H's and T's and total the number of Heads that result during repeated trials.

A far better approach is to use computer graphics to display the results on the number of Heads obtained on each trial, as shown in Figs 3 and 4. This program gives the student a far better appreciation of what is happening than anything that can be done on the blackboard or with numerical displays alone. First of all, it gives a visual feel for randomness both in terms of the different results obtained on successive runs of the program for the same choice of N and also from an examination of the occasional outlier that arises. In addition, this program also gives a feel for the shape of the binomial distribution, and for the effects of different numbers of coins. Moreover, by also providing a numerical summary of the results obtained, the program simultaneously gives a convincing numerical check on the accuracy of the theoretical predictions. Thus, the students can first verify that the experimental results are "close" to those predicted based on probability and yet also see that the expected values are almost never precisely achieved in practice. Finally, a simple modification of the program permits the use of unfair coins having any desired probability of producing Heads. This provides a nice simulation of a more general binomial experiment.

In a different direction, the probabilities involved in the normal distribution can also be displayed graphically by drawing the normal distribution curve for any choice of the parameters mu and sigma; any two x-values selected by the user are then displayed on the graph as vertical lines and the appropriate area between them is shaded in, as shown in Fig. 5. The program then proceeds to calculate the area of this region using a sophisticated quadrature algorithm. This provides the student with a far greater appreciation of the relationship between the probability that X falls between two values (or equivalently that Z falls between the two corresponding z-values) and the area of the corresponding region under the curve. It is no longer a matter of just picking magic numbers out of a table; those numbers now have a visual significance. On the other hand, the use of such a program should not replace the use of the actual tables (at least for the foreseeable future until hand-held calculators have the capability of producing all table entries); but such a graphics program certainly does help students in understanding the significance of the entries in the tables.

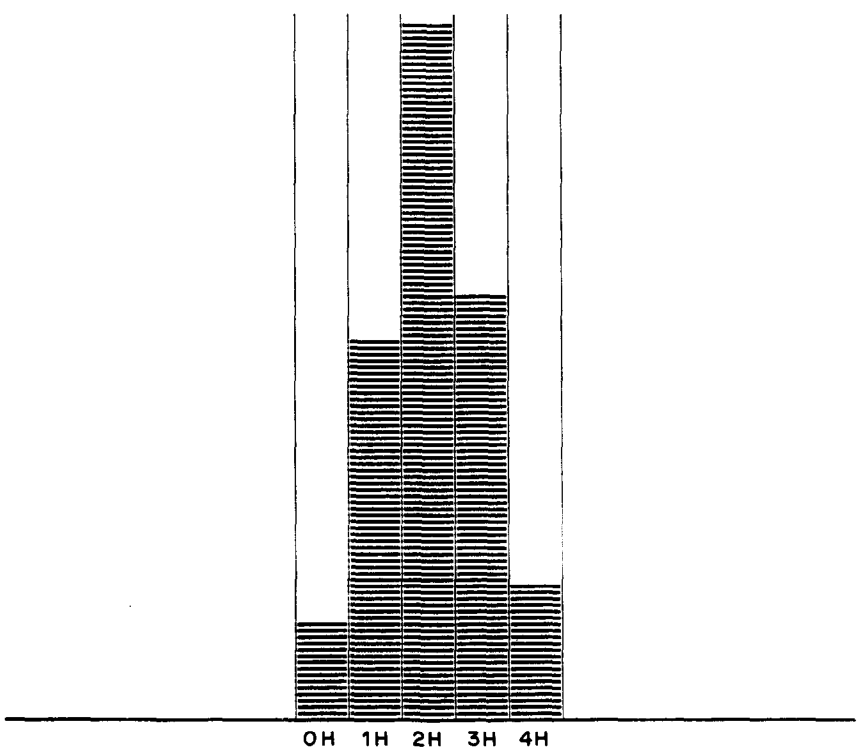

Fig. 3. Binomial simulation: number of heads in 275 flips of 4 coins.

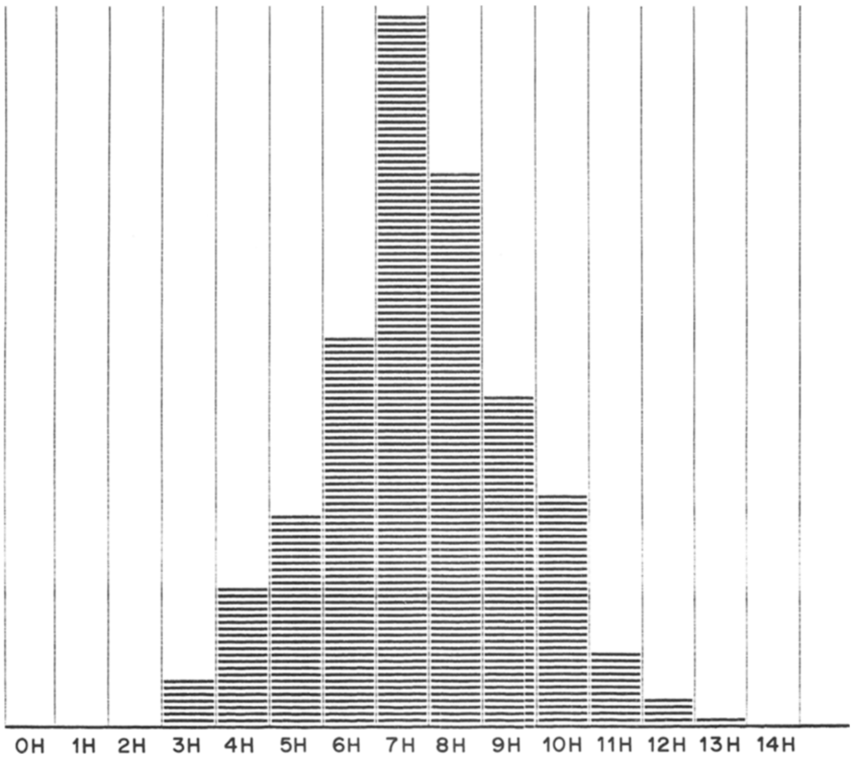

Fig. 4. Binomial simulation: number of heads in 425 flips of 15 coins.

In a similar vein, it is possible to demonstrate graphically the binomial distribution for any choice of probability of success p and number of trials N. With the computer doing all the work in seconds, it is easy for the students to develop an appreciation for the effects of changing either p or N. Further, as shown in Fig. 6, by superimposing the graph of the theoretical normal distribution over

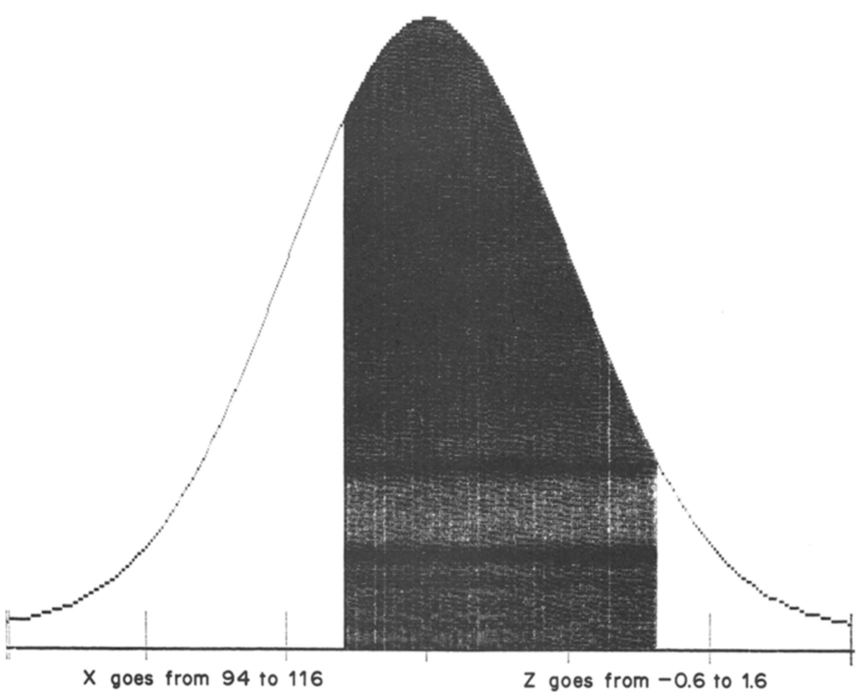

Fig. 5. Probability and area under the normal curve.

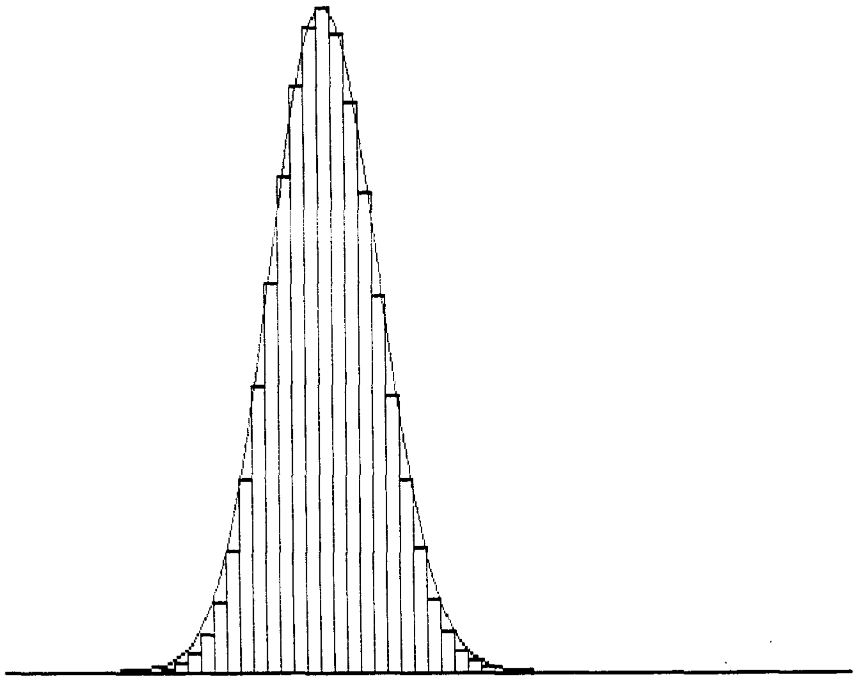

Fig. 6. Binomial distribution histogram and approximating normal. Approximating normal distribution has mean = 24 and S.D. = 3.79, $N = 60$ and $p = 0.4$.

the binomial histogram, it is possible to develop a better understanding of the concept of approximating the binomial distribution with the corresponding normal distribution and for the need for continuity corrections. In a more sophisticated course, such a program also serves to increase student understanding of the concept of convergence.

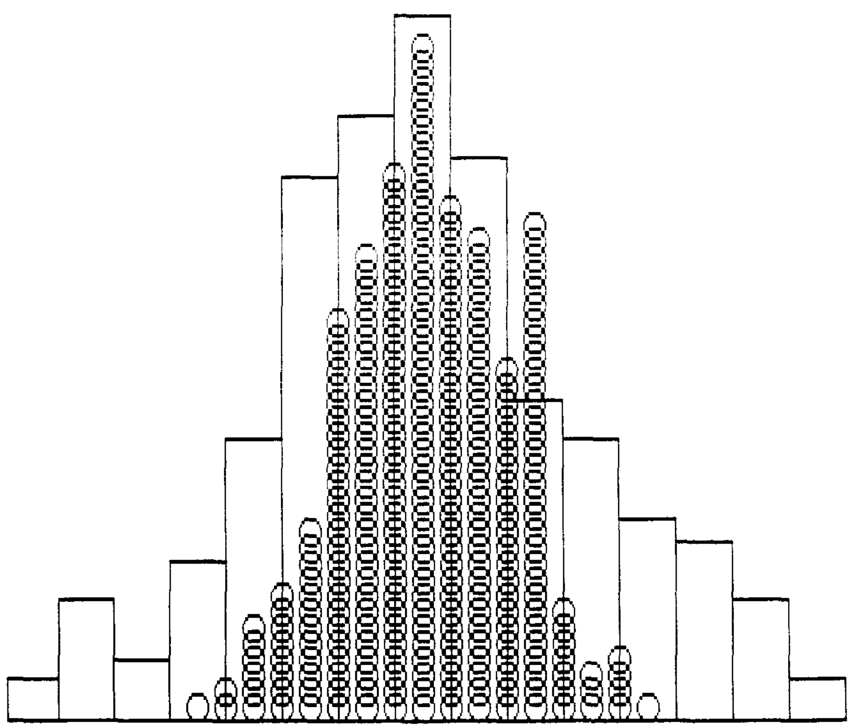

Fig. 7. Central Limit Theorem simulation: means of 290 samples of size 4 from a normal population. Original population: mean = 68.07 and S.D. = 2.76; sampling distribution: $n = 4$ and S.D. = 1.38.

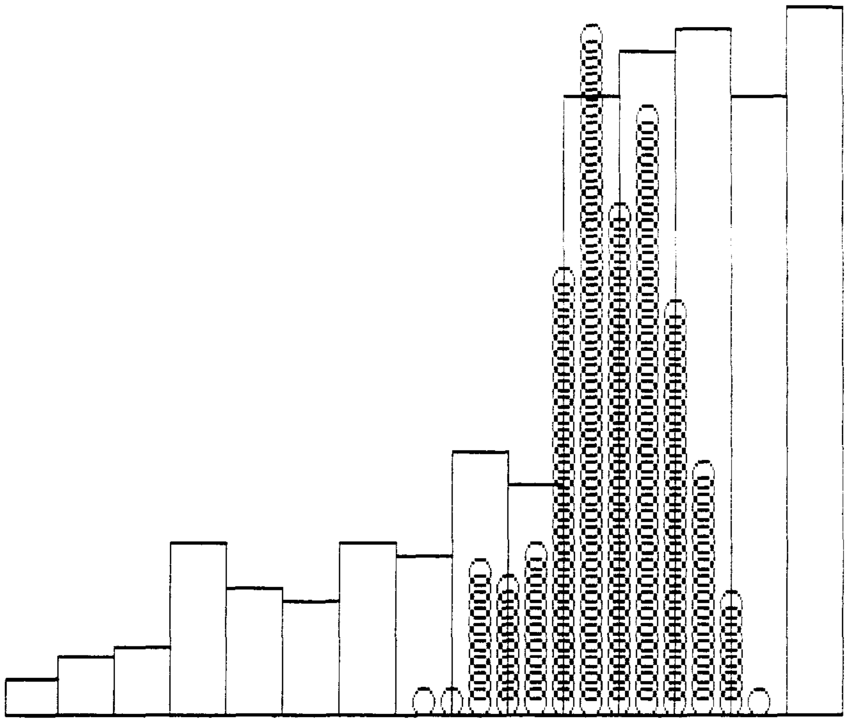

Fig. 8. Central Limit Theorem simulation: means of 218 samples of size 9 from a skewed population. Original population: mean = 71.19 and S.D. = 3.29; sampling distribution: $n = 9$ and S.D. = 1.1.

The Central Limit Theorem, while the primary mathematical tool of inferential statistics, is undeniably the least understood topic in any introductory statistics course. The idea for a graphics simulation-demonstration of this theorem was originally conceived of and developed by Eliot Silverman of Suffolk County Community College for a PDP-11 time-sharing computer system and since adapted by the authors for a variety of microcomputers. In any of these versions, the user has the choice of selecting any of four underlying populations: normal, skew, uniform or u-shaped. He or she then selects the size (2–36) of the desired samples to be chosen and the number of such samples which will be generated. The program first draws the histogram of the chosen underlying population. It then randomly generates the samples, calculates the mean of each sample, and graphs it. The resulting distribution of sample means literally grows in front of the user (each run takes a couple of minutes to complete unless the sample size is rather large). Figure 7 shows the results of using the program with samples of size 4 drawn from a normal population; Fig. 8 shows the results with samples of size 9 from a skewed population; Fig. 9 shows the results with samples of size 36 from a u-shaped population.

When the program is run several successive times using the same underlying population with successively larger values for the sample size, it is extremely simple to demonstrate the convergence of the sampling distribution to the normal distribution as sample size increases. Similarly, the graphic displays clearly show the effects of sample size on the standard deviations. If nothing else, a rough visual estimate of the spread of the sampling distribution can be compared to the width of the full screen to indicate the approximate values of one-half, one-third and one-sixth and so forth. Most students will even anticipate the formula for $\sigma_{\bar{x}}$ from this visual analysis. Further, the program also displays numerical values for the mean and standard deviation of the set of actual samples selected. When these values are compared to the theoretical values predicted by the Central Limit Theorem for the distribution of sample means, it is extremely effective in validating the theory. The improvement in student understanding using this program has been dramatic.

Another effective use of computer graphics in a classroom is in a demonstration of the convergence of the t-distribution to the normal distribution as the sample size increases. The graphs of the t-distribution corresponding to d.f. = 1, 5, 10, 15, 20, 25 and 30 are drawn sequentially and

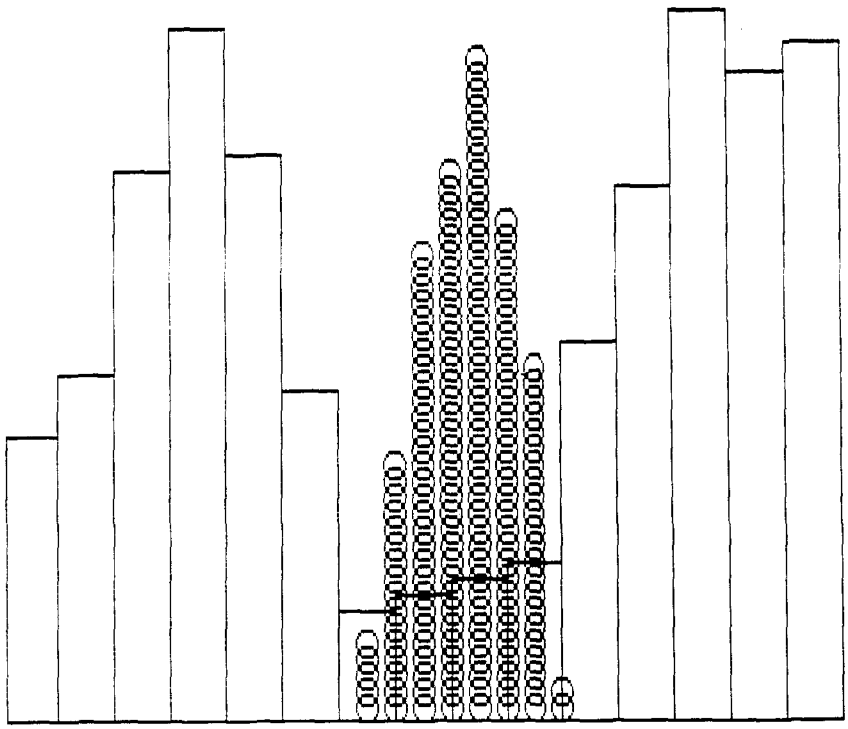

Fig. 9. Central Limit Theorem simulation: means of 183 samples of size 36 from a u-shaped population. Original population: mean = 68.61 and S.D. = 4.8; sampling distribution: n = 36 and S.D. = 0.80.

the convergence to the standard normal curve, also drawn, becomes visually clear to the students, as shown in Fig. 10. Students no longer have to infer the convergence from a table of numerical values. This is something they can see!

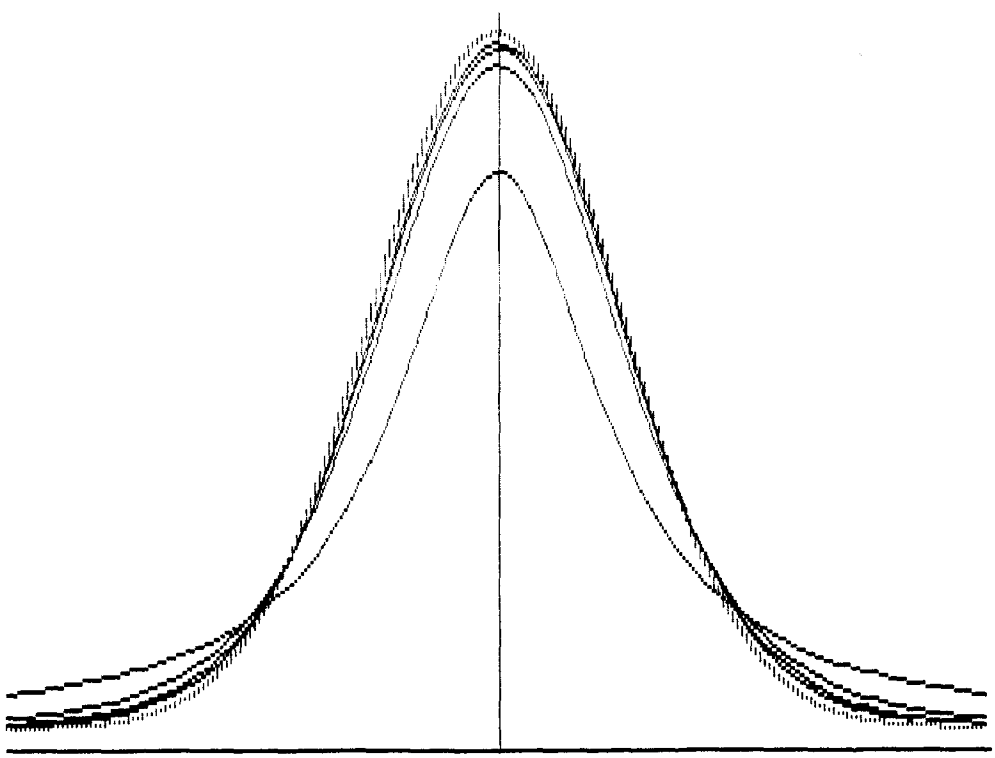

Fig. 10. Convergence of t-distribution (with d.f. = 1, 5, 10, 15) to normal distribution.

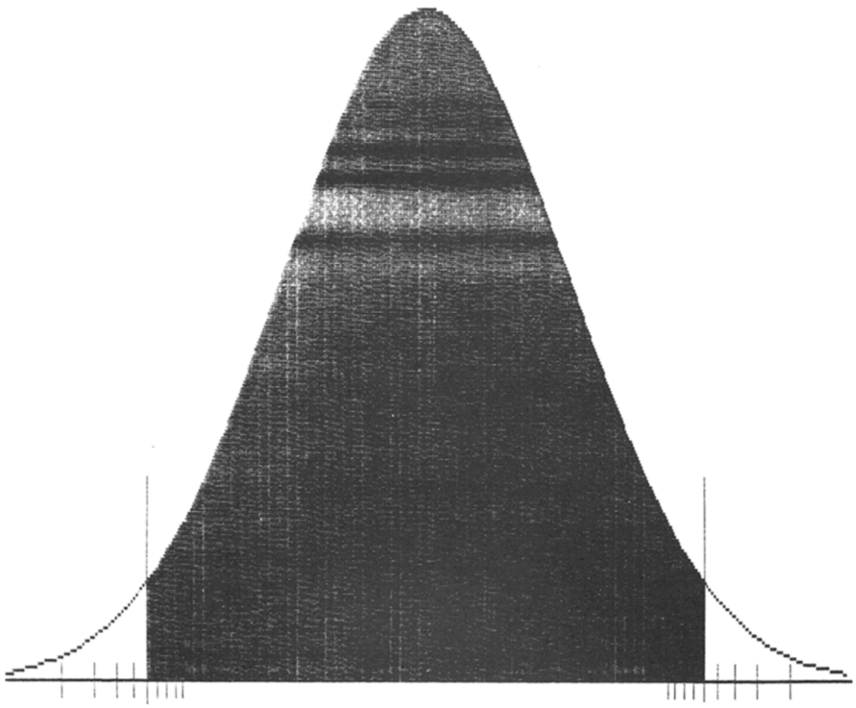

Fig. 11. Graphical display representation of confidence intervals. 95% confidence interval for means.

Estimation via confidence intervals is another standard topic in elementary statistics which students often end up treating mechanically with little comprehension. An appropriate graphics program can do much to improve the level of student understanding. For instance, such a program can lead the user through the preliminary analysis: is the confidence interval for means or porportions? Is the sample size large or small? The program then requests the appropriate input data: mean, standard deviation, sample size and confidence level for means; or sample proportion, sample size and confidence level for proportions. The corresponding graph, which includes tick-marks for all usual confidence levels, as shown in Fig. 11, gives greater insight into the validity of the estimation.

In a similar way, it is possible to generate a graphics demonstration of hypothesis testing. Just as with confidence intervals, the user selects between means and proportions, large and small samples, and inputs the appropriate sample data and level of significance. The program will then draw the graph of the corresponding distribution, indicate the acceptance and rejection regions for the chosen significance level, shade in the rejection region and indicate the point corresponding to the sample data.

Furthermore, the solution to any hypothesis test problem is quite structured in terms of a series of sequential steps and decisions, including statement of the null and alternate hypotheses, selection of one or two tails, type of distribution, critical values and so forth. Many students blithely skip over much of this analysis. A computer program forces them to focus on the necessary steps and develops the corresponding methodology in their work habits.

Still another topic which lends itself to computer graphics is regression analysis. Programs have been written which will draw the scatterplot for any set of input data and the corresponding regression line (see Fig. 12). The speed of the computer allows the student to see the effects of changing, adding or deleting a point on the resulting regression line in seconds, thereby getting a better understanding of the concept of regression. This capability allows the student to turn regression analysis into an experimental activity whereby he or she is able to see the effects, for example, of removing an apparently unrelated data point. Simultaneously, the student is also able to see the effects of altering the data set on the numerical values for the regression coefficients and the correlation coefficient.

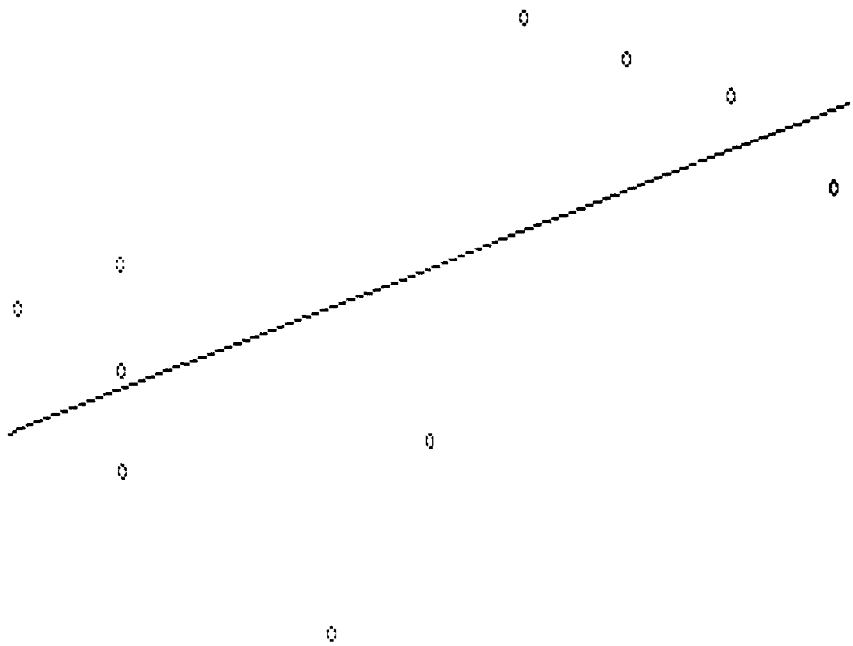

Fig. 12. Display of scatterplot and regression line. The regression line is: $y = 4.599x + 10.243$.

Beyond this, there have also been some efforts at using computer graphics for displays of multivariate data and some of the concepts of multivariate analysis. Most notable among these is a package of statistical routines by Bruce Trumbo which was published by Compress. Especially in such three dimensional situations, the visual display possible using computer graphics is extraordinarily effective.

IMPLICATIONS AND DISCUSSION

In a broader sense, the primary objective in using the computer in education should be as a conceptual tool. We remarked above that most of the current computer usage in statisitics is essentially computational in nature so that the student is freed from the drudgery of doing hand calculations. As a consequence, there is a considerable saving in the amount of class time previously devoted to such calculations. In turn, this presents the instructor with the opportunity to utilize the extra time in a variety of ways.

Probably the most common way in which this extra time is spent is by dealing with considerably more complicated problems—for instance, using larger samples. This certainly gives the student a greater appreciation for the versatility and applicability of statistics in real world settings. The other primary use made of the extra time is to devote it to covering additional topics. This gives the student a larger assortment of statistical tools and a better appreciation of the statistical methods. Unfortunately, neither one of these approaches addresses what the authors believe to be the most desirable outcome—increasing student understanding.

Certainly, of the various possibilities for spending the extra time, the third, improving understanding, is by far the hardest to implement. Most standard introductory textbooks do not emphasize statistics as an experimental endeavor in which the student sees the effects of changing data or parameters. That is virtually impossible to do effectively without a computer. Rather, the books give an impression that statistics is simply a collection of standardized methods that the student must learn to apply. As a consequence, most students come out of such a course with little understanding of statistical methods and quickly forget how to apply them. Covering more complicated problems or a greater variety of topics simply compounds this problem rather than solves it.

Fortunately, the computer, particularly through its graphics capabilities, now provides the opportunity for an instructor to change the emphasis in introductory statistics courses and focus more on an understanding of the underlying statistical concepts. Hopefully, this will result in a more lasting appreciation of statistics on the part of the students and will go a long way towards achieving the goal of universal statistical literacy among college graduates.

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